How to Use Coefficient Tables for Computerized Celestial Position Calculations

1. Compute the Day Number T

First, determine the day number T from the Universal Time date for which the celestial body is to be calculated.

$T = 30 \times P + Q (S - Y) + P (1 - Q) + Day$

Where:
$P = Month - 1$
$Q = [(Month + 7) / 10]$
$Y = [(Year / 4) - [Year / 4] + 0.77]$
$S = [P \times 0.55 - 0.33]$

The symbol [ ] denotes the Gaussian integer function (floor function), meaning the greatest integer not exceeding the enclosed value.

Examples:
$[3.95] = 3$
$[-2.05] = -3$

2. Express the Observation Time as the Fraction of a Day F

The observation time (Universal Time) is expressed as the fractional part of a day:

$F = (Hour / 24) + (Minute / 1440) + (Second / 86400)$

3. Add ΔT to Obtain t

$t = T + F + \Delta T / 86400$

For the year 2027, this publication assumes:
$\Delta T = 69.184$ seconds

This ΔT is a correction applied in order to maintain a uniformly flowing time scale for celestial coordinate calculations. Therefore, it must always be treated as a fixed value with no short-term fluctuations.

The only time this value changes is when a leap second is added or removed.

This is because the calculations are not based on the UT1 standard used for Earth rotation angle computations.

Furthermore, Barycentric Dynamical Time (TDB) has already been taken into account internally, so there is no need to calculate it separately.

4. Compute R.A., Declination, Distance, and Horizontal Parallax

For each celestial body, coefficients are provided for the calculation of:

- Right Ascension (R.A.)
- Declination (Dec.)

Additionally:
- The Sun and planets include coefficients for geocentric distance (Dist.)
- The Moon includes coefficients for horizontal parallax (H.P.)

To obtain these values, first compute the argument θ:

$\theta = \mathrm{Cos}^{-1}((\ 2t - (\ a + b\ )\ )\ /\ (\ b - a\ ))$

Then compute the desired quantity f(t) using:

$f(t) = C_0 + C_1 \cos\theta + C_2 \cos 2\theta + \ldots + C_N \cos N\theta$

Although the coefficient tables contain many terms (typically 0–40), users may freely adjust the number of terms according to the required precision.
For example, if extremely high precision is unnecessary, calculations may be performed using only terms 0–20.

5. Compute the Greenwich Hour Angle (G.H.A.)

h = E + UT
E = R – R.A.

When calculating R, use:
t = T + F

without applying ΔT.

6. Compute the Apparent Semidiameter

Sun:
Compute the geocentric distance Dist. at time t:

S.D. = 16’.02 / Dist.

Moon:
Using the horizontal parallax H.P.:

$$S.D. = \sin^{-1}(0.2725 \sin H.P.)$$

Planets:
S.D. = S_0 / Dist.

Planetary constants S_0:
Venus: 8.3”
Mars: 4.7”
Jupiter: 92.1” (polar radius), 98.4” (equatorial radius)
Saturn: 73.8” (polar radius), 82.7” (equatorial radius)

7. Calculation of Lunar Libration Seen from the Sun

The calculation of the Moon’s subsolar latitude, longitude, and orientation is somewhat special.

The subsolar longitude Ls is internally treated as a value exceeding 360° in order to maintain numerical continuity during the creation of the coefficient equations.

However, the actual longitude is normally expressed within the range:
-180° to 180°

Therefore, users should perform the following procedure:

1. Compute Ls normally.
2. If Ls exceeds 360°, reduce it into the 0°–360° range using a MOD function.
3. After Step 2, if the value exceeds 180°, subtract 360° to convert it into the corresponding negative value.

By following this procedure, the correct subsolar lunar longitude Ls can be obtained.

The orientation angle Ps of the Moon as seen from the Sun can then be calculated using the following equation.

$$Tan(Ps) = \frac{Cos(Sun\ R.A.) \times Sin(Sun\ R.A. - Moon\ R.A.)}{Sin(Sun\ Dec.) \times Cos(Moon\ Dec.) - Cos(Sun\ Dec.) \times Sin(Moon\ Dec.) \times Cos(Moon\ R.A. - Sun\ R.A.)}$$

where:

- Sun R.A. = apparent right ascension of the Sun
- Moon R.A. = apparent right ascension of the Moon
- Sun Dec. = apparent declination of the Sun
- Moon Dec. = apparent declination of the Moon

Determine the Sun's E, d, h, and S.D. for July 21 2027, at 11:15:13 Universal Time.

For simplification, only the first 20 coefficient terms (0–20) are used in this calculation.

P = 5 - 1 = 4

Q = [(12) / 10] = 1

Y = [(2027 / 4) - [2027 / 4] + 0.77] = 1

S = [4 × 0.55 - 0.33] = 1

T = 30 × 4 + 1 × (1 - 1) + 4 × (1 - 1) + 21 = 141

F = (11 / 24) + (15 / 1440) + (13 / 86400) = 0.4689005

t = 141 + 0.4689005 + 69.184 / 86400 = 141.4697012

The values of a and b satisfying a ≤ t ≤ b correspond to the interval April 30 – September 1, a = 120 , b = 244

θ = Cos⁻¹((2 × 141.4697012 - (244 + 120)) / (244 - 120)) = 130.822248

Using this value of θ together with the coefficients, compute the apparent right ascension, apparent declination, _ and geocentric distance.

In some cases, the values of R.A. or R may become less than 0h or greater than 24h. In such cases, simply add or _ subtract 24h as necessary.

N	Nθ	CosNθ	R.A. C NCosNθ h	Dec. C NCosNθ °	Dist. C NCosNθ AU
0	0.00000000	1.00000000	6.60431692	17.17443434	1.01235971
1	130.82224801	-0.65371450	-2.70379836	2.15773415	-0.00078056
2	261.64449603	-0.14531471	0.00597599	0.84191926	0.00060846
3	32.46674404	0.84370317	-0.03442011	0.17704414	-0.00004458
4	163.28899205	-0.95776727	-0.00499175	-0.15340471	-0.00010036
5	294.11124006	0.40850953	0.00133984	-0.00429829	0.00000280
6	64.93348808	0.42367007	-0.00018200	-0.00213362	0.00000134
7	195.75573609	-0.96242806	0.00012250	-0.00065684	-0.00000849
8	326.57798410	0.83463628	0.00004992	0.00002928	-0.00000710
9	97.40023211	-0.12879961	-0.00000158	0.00002708	-0.00000013
10	228.22248013	-0.66623993	-0.00002034	-0.00007294	0.00000677
11	359.04472814	0.99986101	-0.00005651	0.00010382	-0.00001068
12	129.86697615	-0.64100735	0.00002571	-0.00010165	-0.00000862
13	260.68922416	-0.16178942	-0.00000681	0.00000027	-0.00000119
14	31.51147218	0.85253553	0.00001562	-0.00013694	-0.00000566
15	162.33372019	-0.95284025	0.00001603	0.00000266	0.00000290
16	293.15596820	0.39323544	-0.00000306	0.00002617	0.00000074
17	63.97821622	0.43871284	0.00000237	0.00000249	0.00000035
18	194.80046423	-0.96682132	-0.00000117	0.00001788	0.00000035
19	325.62271224	0.82533739	-0.00000059	-0.00000926	0.00000015
20	96.44496025	-0.11224871	-0.00000025	-0.00000079	-0.00000001
Sum			3.868382	20.190526	1.012016
		NAOJ Calculated Value	3.868381	20.190513	1.012017
		Difference	0.000001	0.000013	0.000001

As shown above, very high accuracy can be achieved even when only a limited number of coefficient terms are used.

Next, determine the value of R.
For the calculation of R, the value of t does not include ΔT. Therefore,

$t = 141.4689005$

$\theta = 130.8232259$

N	Nθ	CosNθ	C^{R}_{N}CosNθ h
0	0.0000000	1.0000000	18.5884979
1	130.8232259	-0.6537274	-2.6633134
2	261.6464518	-0.1452809	0.0000006
3	32.4696776	0.8436757	-0.0000045
4	163.2929035	-0.9577869	-0.0000013
5	294.1161294	0.4085874	0.0000003
6	64.9393553	0.4235773	0.0000005
7	195.7625811	-0.9623956	0.0000000
8	326.5858070	0.8347115	0.0000012
9	97.4090329	-0.1289519	0.0000001
10	228.2322588	-0.6661126	-0.0000003
Sum			15.9251810
NAOJ Calculated Value(Sidereal Time - Universal Time)			15.9251786
		Difference	0.0000024

Therefore, the value of R at the observation time becomes 15.925181h
E = R - R.A. =12.056799h
that is, 12h03m24s

As obtained above, the apparent declination is 20.190526°, that is, 20°11.'4N

$$S.D = \frac{16.02}{1.012016} = 15.8297893$$

that is, 15' 49.8" , demonstrating very high computational accuracy.

The Greenwich Hour Angle (G.H.A.) at the observation time is obtained as
G.H.A. = E + U.T. = 23h18m37s

Because the calculation of the Moon's subsolar latitude, longitude, and orientation as seen from the Sun requires _ special treatment, an example calculation is presented below.
Determine the lunar libration angles Bs, Ls, and Ps as seen from the Sun for May, 21, 2027, at 0h0m0s U.T.
For simplification, only the first 20 coefficient terms (0–20) are used in this calculation.

P = 5 - 1 = 4

Q = [(5 + 7) / 10] = 1

Y = [(2027 / 4) - [2027 / 4] + 0.77] = 1

S = [4 × 0.55 - 0.33] = 1

T = 30 × 4 + 1 × (1 - 1) + 4 × (1 - 1) + 21 = 141

F = (11 / 24) + (15 / 1440) + (13 / 86400) = 0.4689005

t = 141 + 0.4689005 + 69.184 / 86400 = 141.4697012

The values of a and b satisfying a ≤ t ≤ b correspond to the intervalApril 30 – June 1 a = 120 , b = 152

$\theta = \mathrm{Cos}^{-1}((2 \times 141.4697012 - (152 + 120)) / (152 - 120)) = 70.0099881$

Using this value of θ and the coefficient tables, compute Bs, Ls.

N	Nθ	CosNθ	Bs C NCosNθ °	Ls C NCosNθ °
0	0.00000000	1.00000000	1.49441941	418.77602282
1	70.00998815	0.34185633	-0.02081188	-66.78288267
2	140.01997629	-0.76626851	0.02073079	-0.06758726
3	210.02996444	-0.86576380	0.01793220	0.04853719
4	280.03995259	0.17433485	-0.00002495	-0.00795215
5	350.04994073	0.98495874	0.00634638	0.01374577
6	60.05992888	0.49909390	0.00015413	0.00240843
7	130.06991703	-0.64372192	0.00112650	0.00165152
8	200.07990517	-0.93921472	0.00007993	0.00004862
9	270.08989332	0.00156893	0.00000068	0.00000060
10	340.09988147	0.94028742	-0.00002706	-0.00007448
11	50.10986961	0.64131747	-0.00005271	-0.00000939
12	120.11985776	-0.50181055	-0.00001008	-0.00000947
13	190.12984591	-0.98441170	-0.00001015	0.00000989
14	260.13983405	-0.17124418	0.00000093	0.00000010
15	330.14982220	0.86732989	0.00000006	0.00000285
16	40.15981035	0.76424859	0.00000053	-0.00000079
17	110.16979849	-0.34480346	0.00000014	0.00000017
18	180.17978664	-0.99999508	-0.00000009	-0.00000039
19	250.18977479	-0.33890583	-0.00000003	0.00000000
20	320.19976293	0.76828088	-0.00000006	-0.00000005

Sum	1.52	351.9839113
In Excel, the value can be reduced into the 0° - 360° range using		351.98
Furthermore, if the value exceeds 180° , subtract 360° to convert it into _ a negative value.		-8.02
NAOJ Calculated Value	1.52	-8.02
Difference	0.00	0.00

The orientation of the lunar subsolar point Xs as seen from the Sun can then be calculated using the equation _ given above, as follows.
The required arguments may be obtained from the present calculation equations or computed separately by the user.

Sun_dec(Solar declination) = 20.1905136 Sun_R.A.(Solar right ascension) = 58.0257116
Moon_dec(Lunar declination) = -26.6289969 Moon_R.A.(Lunar right ascension) = 249.7182733

$$\mathrm{Tan}(Ps) = \frac{\mathrm{Cos}(20.1905136) \times \mathrm{Sin}(58.0257116 - 249.7182733)}{\mathrm{Sin}(20.1905136) \times \mathrm{Cos}(-26.6289969) - \mathrm{Cos}(20.1905136) \times \mathrm{Sin}(-26.6289969) \times \mathrm{Cos}(249.7182733 - 58.0257116)}$$

Ps =	118.53	
	118.53	NAOJ Calculated Value
	0.00	Difference